Basics

In order to be able to draw conclusions with regard to the reliability of a component or assembly, tests are conducted with a limited number of test samples prior to actual series production. This is a relatively reliable method of discovering fundamental design flaws or manufacturing faults. On the other hand, the probability of determining faults that occur randomly or at low frequency is low if a considerably higher load cannot be applied in the test. This is generally the case in vehicle tests, in contract to the special component tests conducted on component test rigs or in the laboratory, permitting an increase in load by a factor of 2 and higher.
The initial question is how high is the probability P_A that a test specimen fails during the test:

$P_A = 1 - R_t^n$ where R_t = Reliaiblity at test time t for a test specimen; n = Number of test specimen

Rearranging
the formula:

$$R_t = (1 - P_A)^{1/n}$$

The reliability for the test time t is calculated using:

$$R_t = e^{-(t/T)^b}$$

A reliability R_a: applies for the defined service life t_a:

$$R_a = e^{-(ta/T)^b}$$

Equating the two relationships and defining $Lv = t / t_a$ results in:

$$\frac{R_t}{R_a} = \frac{e^{-(t/T)^b}}{e^{-(ta/T)^b}} \quad -> \quad \frac{\ln(R_t)}{\ln(R_a)} = \frac{-(t/T)^b}{-(ta/T)^b} = Lv^b$$

Consequently:

$$\ln(R_t) = \ln(R_a)\, Lv^b$$

$$R_t = R_a^{Lv^b}$$

Together with the number of test specimens $R_t = (1 - P_A)^{1/n}$ and equating results in:

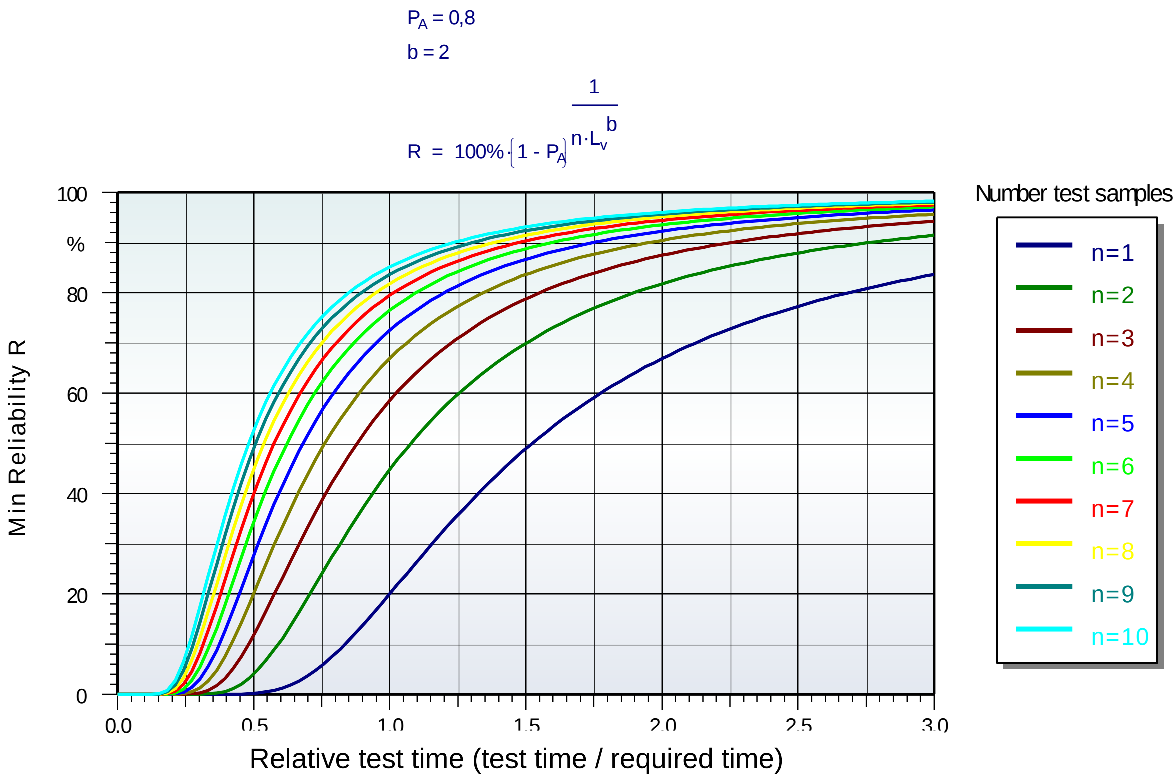
$$R_t = R_a^{L_v^b} = (1 - P_A)^{\frac{1}{n}}$$

$$R_a = (1 - P_A)^{\frac{1}{n\, L_v^b}}$$

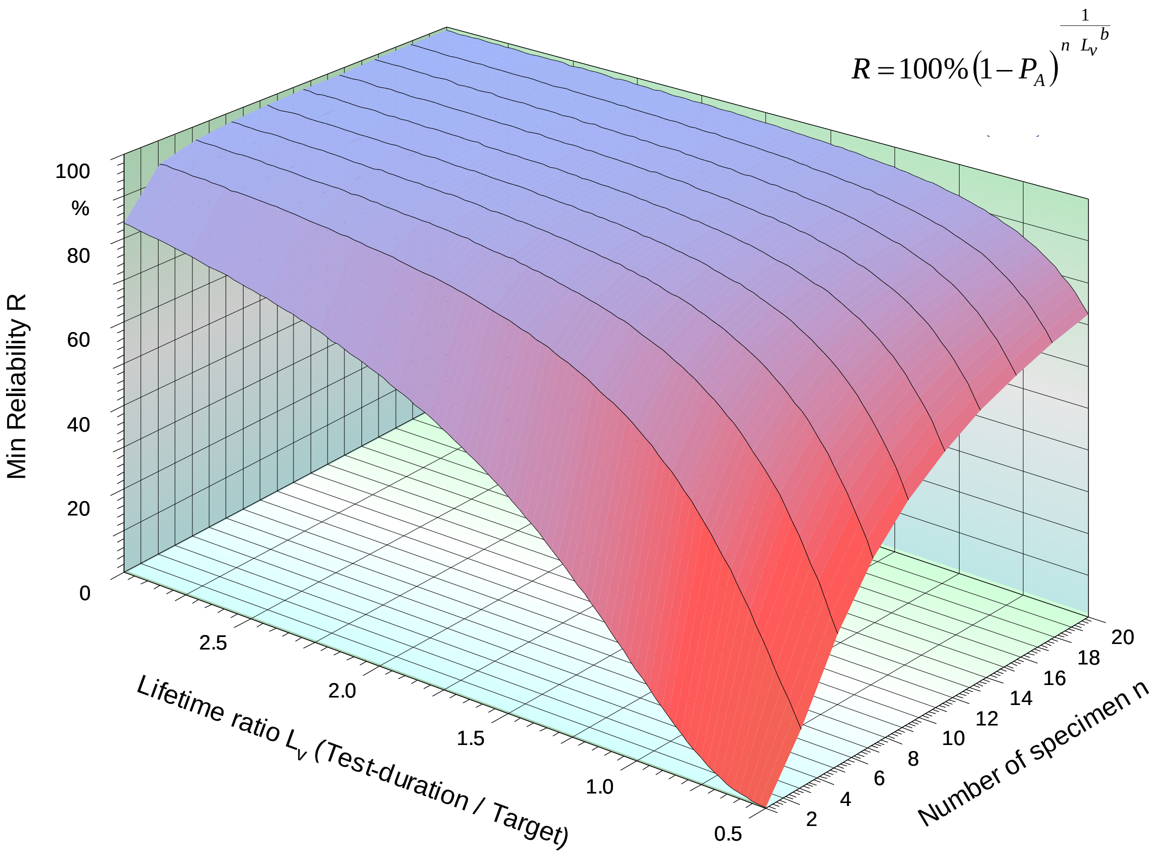
The reliability R_a is to be considered as the "guaranteed minimum reliability“ and the following applies:

$$R_{min} = (1 - P_A)^{\frac{1}{n\, L_v^b}}$$

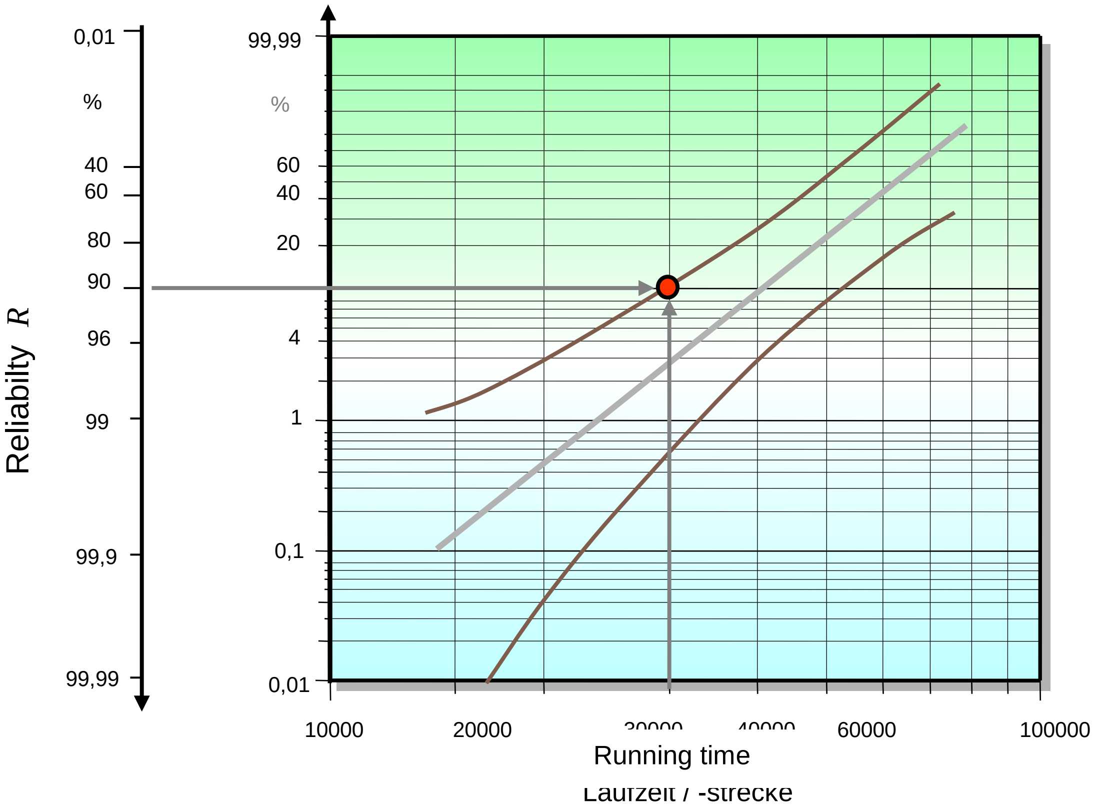
Example: The following representation is achieved for a confidence level P_A=0.8 and an estimated b=2:



In the 3D representation it can be seen that a further increase at the level of L_v and n provides no further decisive advantages.



On examining these considerations in the Weibull plot the following representation is obtained (P_A = 0.80 > upper confidence bound 90% t = 30000 R_{min} = 90%)



P_A was originally introduced as the probability for the failure of a test specimen. The prerequisite is that these test specimens are a random sample of a "population". A conclusion is drawn based on the samples where the definition corresponds to the confidence bound of the illustrated Weibull plot.

! The calculated minimum reliability is not valid if the test specimens are "hand specimens" or prototypes with their manufacturing process not corresponding to subsequent series production.

! It should be noted that a lower b results in a lower minimum reliability. Initially, this is not to be expected as a low b results in a lower slope on the Weibull plot and therefore a higher failure frequency. This effect is caused if the target running time is less than the test time and one is moving to the left with a flat slope.

It can generally be assumed that for the confidence level or for determining the reliability, it is better to test less samples size for a longer time than many samples for a relatively short test time. On the other hand, with fewer samples the conclusion concerning the component scatter is also less reliable (minimum number of samples).

! Lv should always be greater than 1 if the load cannot be increased. Irrespective of the mathematical minimum reliability, no part must fail at Lv<1 (minimum requirement).

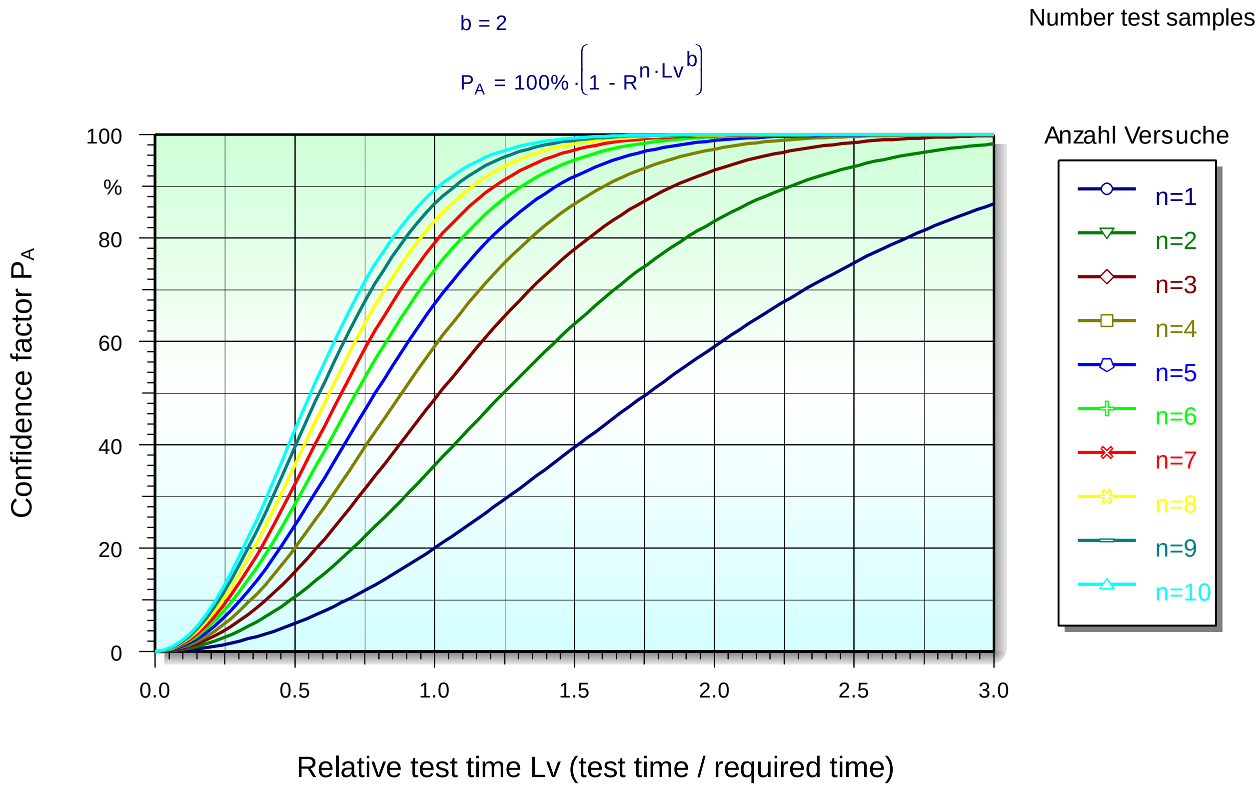
If conclusions are to be drawn with regard to the reduction in service life to higher load, tests with concrete failures will be necessary, represented in a stress-cycle (Woehler) diagram.

If a certain minimum reliability is defined and the question is what confidence level is reached, the above formula is to be correspondingly rearranged to result in b=2 and R=80%:

$R = 0{,}8$

$b = 2$

$$P_A = 100\% \cdot \left(1 - R^{n \cdot Lv^b}\right)$$

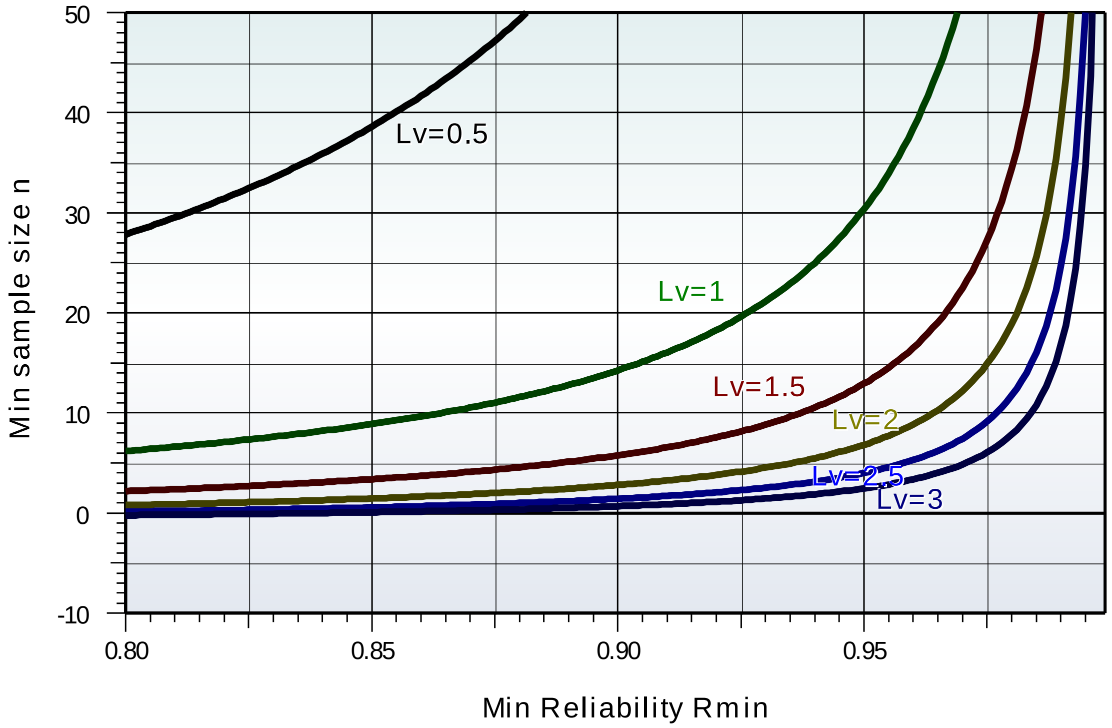


Minimum number of samples for tests

It is often necessary to determine the minimum number of test specimen for the purpose of verifying the defined reliability. However, there is no generally defined procedure for this purpose. In accordance with VDA, the necessary minimum number is calculated by transposing the formula for n:

$$n = \frac{1}{L_v^b}\left[\frac{\ln(1-P_A)}{\ln(R)}\right]$$

For P_A=0.80 and b=2 this results in:



As illustrated, the prerequisite for this scope of random samples is that no failures occur.
The procedure for determining the confidence bound can be used for establishing the minimum number of samples, resulting in the same consideration as when using a defined confidence level.

Example: The number of components to be tested is to be found if a double test time compared to the required service life is possible and a minimum reliability of R=90% is required. No parts fail during the test. This results in n=3 for a confidence level of P_A=0.80.

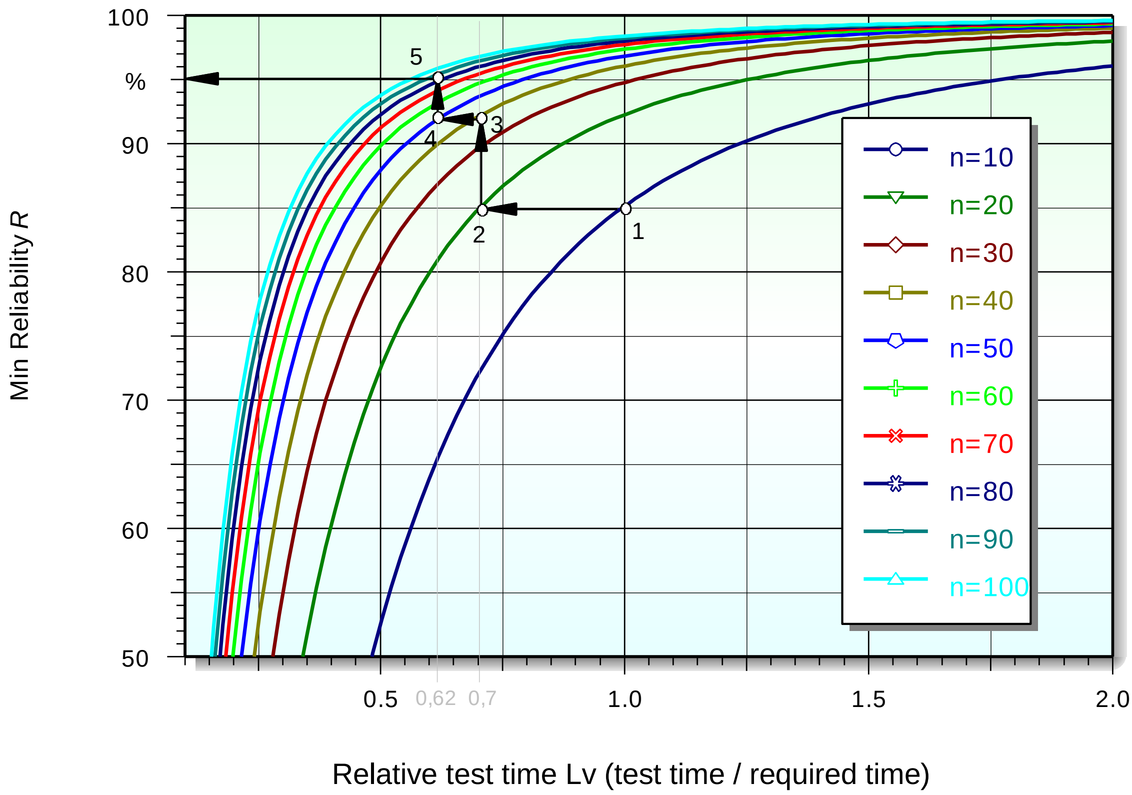
Determining minimum reliability for several test groups with different running times

If there are several identical products with different running times used in the test (or in the field), each running time completed without failure will contribute to deducing the minimum reliability. Corresponding classifications of the running time are preferable formed for this purpose. Example: The following running times and number of "test specimens" serve as the basis for a confidence level of P_A=80% and a required service life of 100,000 km (assumption b=2):

i	Running time/km	Lv	n
1	100000	1.0	10
2	70000	0.7	20
3	62000	0.62	40

The running times were sorted in descending order and the calculation started at the longest running time. This produces the following points in the diagram:

1) 10 parts survived without failing at the longest running time of L_v=1.0
2) This corresponds to a quantity of 20 parts at L_v=0.7 (identical R_{min}).
   20 parts were tested without failure at L_v=0.7
3) Together this results in approx. 40 parts at L_v=0.7
4) This corresponds to a quantity of 50 parts at L_v=0.62 (identical R_{min}).
   20 parts were tested without failure at L_v=0.62
5) Together this results in approx. 90 parts at L_v=0.62



The result is a guaranteed minimum reliability of approx. 95%. Referred to the minimum reliability relationship already introduced, the total $R_{min,ges}$ is generally derived from:

$$R_{\min,ges} = 100\% \left(1 - P_A\right)^{\left(\sum_{i=1}^{k} Lv_i^b \, n_i\right)^{-1}}$$

k = Number of different test times (collectiv)

If failures occurred unexpectedly during the tests, the minimum reliability will then be based on the test running time achieved up to this point and the number of test specimens n' still to be tested:

$$R_{\min,ges} = 100\% \left(1 - P_A\right)^{\left(\sum_{i=1}^{k} Lv_i^b + n' \, Lv'^{\,b}\right)^{-1}}$$

k = Number of different test times (collectiv)
n' = Number of test specimens to be tested without failure
Lv' = Test time to be tested for the test specimens without failure

Taking into account previous knowledge

If previous knowledge of the components is available (Bayes method), it can be taken into account by using the Beyer/Lauster method /23/. This previous knowledge can originate, for example, from predecessor models and is expressed by the value R_o that is valid for a confidence level of P_A=63.2%. The expected minimum reliability is:

$$R_{\min} = \left(1 - P_A\right)^{\frac{1}{n\,L_v^b + 1/\ln(1/Ro)}}$$

In the same way as the factor ϕ defined under /26/ for taking into consideration the applicability of the previous knowledge, it is used here under the term previous confidence level to give:

$$R_{\min} = \left(1 - P_A\right)^{\frac{1}{n\,L_v^b + \phi/\ln(1/Ro)}}$$

The previous information factor ϕ must lie between 0…1. ϕ =0 signifies that no previous information should be used whereas ϕ = 1 means all previous information can be used.
ϕ can, for example, assume the following values when the following applies to the components of the earlier tests:

$\phi = 1$	The components and the tests are identical to the current status or are 100% comparable
$\phi = 0.75$	Components have been slightly modified or the design status is identical but from different manufacturers
$\phi = 0.50$	Components have been partially modified, e.g. material properties

$\phi = 0.25$ Components agree only in terms of their concept (rough estimation)

The preliminary confidence level can also be used to express when the test changed. The reduced number of samples is therefore:

$$n = \frac{1}{L_v^b}\left[\frac{\ln(1-P_A)}{\ln(R_{\min})} - \frac{\phi}{\ln(1/R_o)}\right]$$

An acceleration factor can be used to take into account different loads from earlier tests. This acceleration factor is discussed in the following sections dealing with the component strength (service life in the Woehler diagram).

Determining t_{10} (B_{10}) from minimum reliability without failures

If there are no failures in the tests, the following steps can be applied to calculate a t_{10} service life:

Step 1: Determine a minimum reliability R_{min} from existing tests -> Point (1) in the diagram.

Step 2: Determine a mean service life ratio L_{vm}, that is equivalent to the previous tests. The following formula is used for this purpose:

$$R_{\min} = (1-P_A)^{\left(\sum_{i=1}^{k} Lv_i^b\, n_i\right)^{-1}} = (1-P_A)^{\left(Lvm^b\, n\right)^{-1}}$$

Rearranging the right side for L_v results in:

$$L_{vm} = \left(\frac{1}{n}\left(\frac{\ln(1-PA)}{\ln(R_{\min})}\right)\right)^{1/b}$$

Step 3: Calculate the reliability value on the Weibull curve with P_A = 50% -> Point (2)

$$R_{PA=50\%} = (1-0{,}5)^{1/(L_{vm}^b\, n)}$$

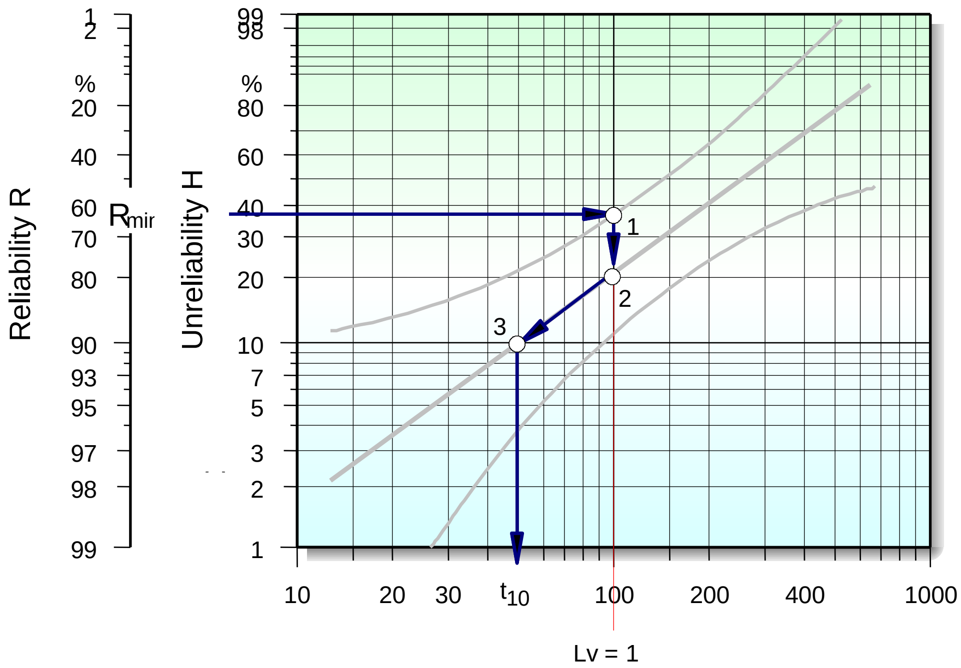
The Weibull curve is now defined by specifying the slope b and the Point (2) on the curve.

Step 4:
Rearranging the Weibull distribution for T results in:

$$T = \frac{t}{(-\ln(1-H))^{1/b}} = \frac{t_{gefordert}}{(-\ln(R_{PA=50\%}))^{1/b}}$$

Step 5:
Calculation of t_{10} (or B_{10}) -> Point (3)

$$t_{10} = T\left(\ln\left(\frac{1}{1-0{,}1}\right)\right)^{1/b}$$



Minimum reliability in tests with unexpected failures

The relationship used to date no longer applies if failures are permitted to occur in the test. R_{min} will then be calculated based on the χ^2 distribution:

$$R_{\min} = e^{-\frac{\chi^2_{2r+2,P_A}}{2 \cdot L_V^b \cdot n}}$$

with r = number of failures during the test

Strictly speaking, the previous example with a failure rate at Lv=1.1 is not quite correct for the further calculation. To simplify matters, it was assumed that the test specimen was removed just before reaching the failure at Lv=1.1.

By way of transposition, the necessary new scope of samples is derived from

$$n = -\frac{\chi^2_{2r+2,P_A}}{2 \cdot L_v^b \cdot \ln(R_{\min})}$$

or the necessary testing time

$$L_v = \left(-\frac{\chi^2_{2r+2,P_A}}{2 \cdot n \cdot \ln(R_{\min})} \right)^{1/b}$$

This approach is to be applied when the times at which the failures occur are still uncertain. However, since the number of failures cannot be forecast in advance in practical terms, this calculation is of corresponding significance only for presenting scenarios.

Example: n=5, b=2, R_{min}=0.8, P_A=0.9
Necessary testing time for the required minimum reliability

No failure Lv = 1.43
1 failure Lv = 1.87
2 failures Lv = 2.18

Reliability from Binomial-method

In general applies to the statistical assurance without given running time the Binomial-method.

$$P_A = 1 - \sum_{i=0}^{x} \binom{n}{i} (1-R)^i \, R^{n-i}$$

with x = number of failures
n = sample size

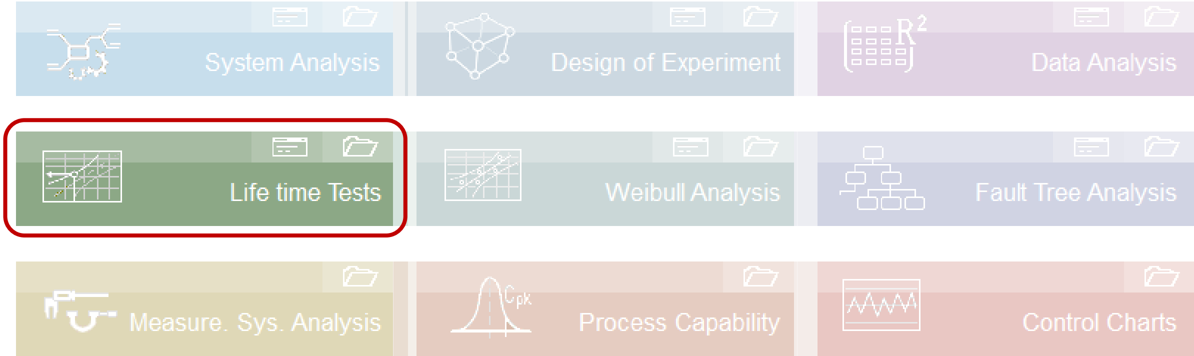
R is the the reliability for not defined running time (normally to describe the quality after production). For the representation often the so called “Larson-Nomongram” is used, because the formula can not be resolved for R. Especially in industrial series production the Binomial-method represents an important tool for assessing the quality level for the sampling technique and for the control charts.

In case of no failures (x = 0) the equation becomes the simple form

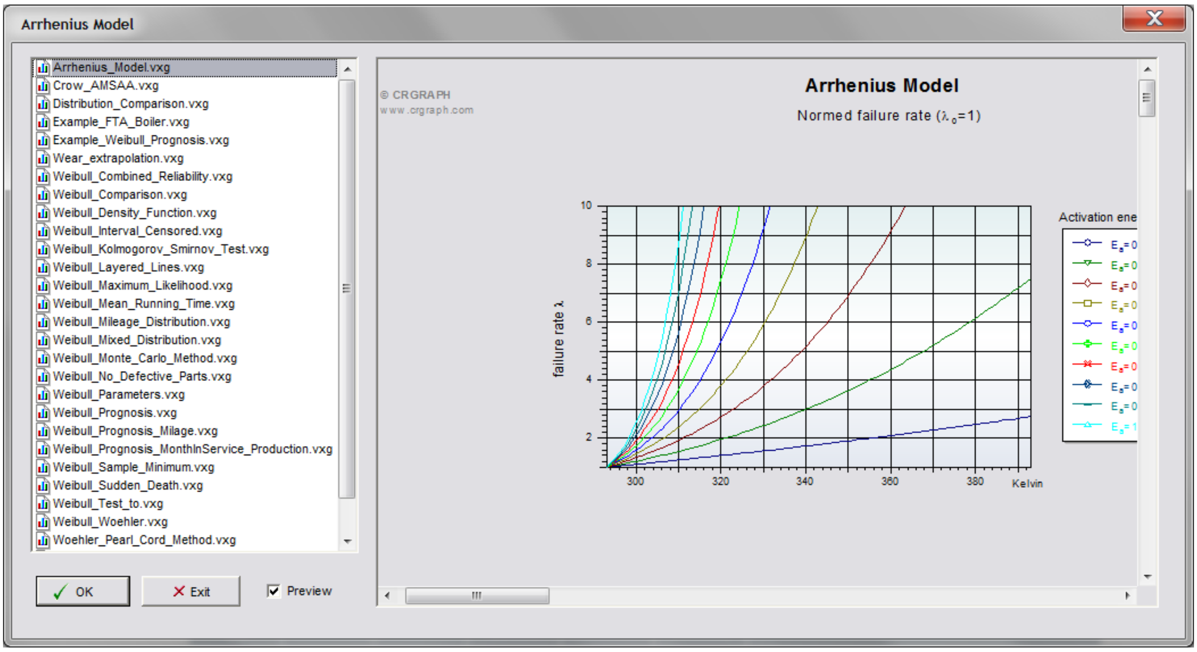
$$P_A = 1 - R^n$$

which is conform to the success-run-method.

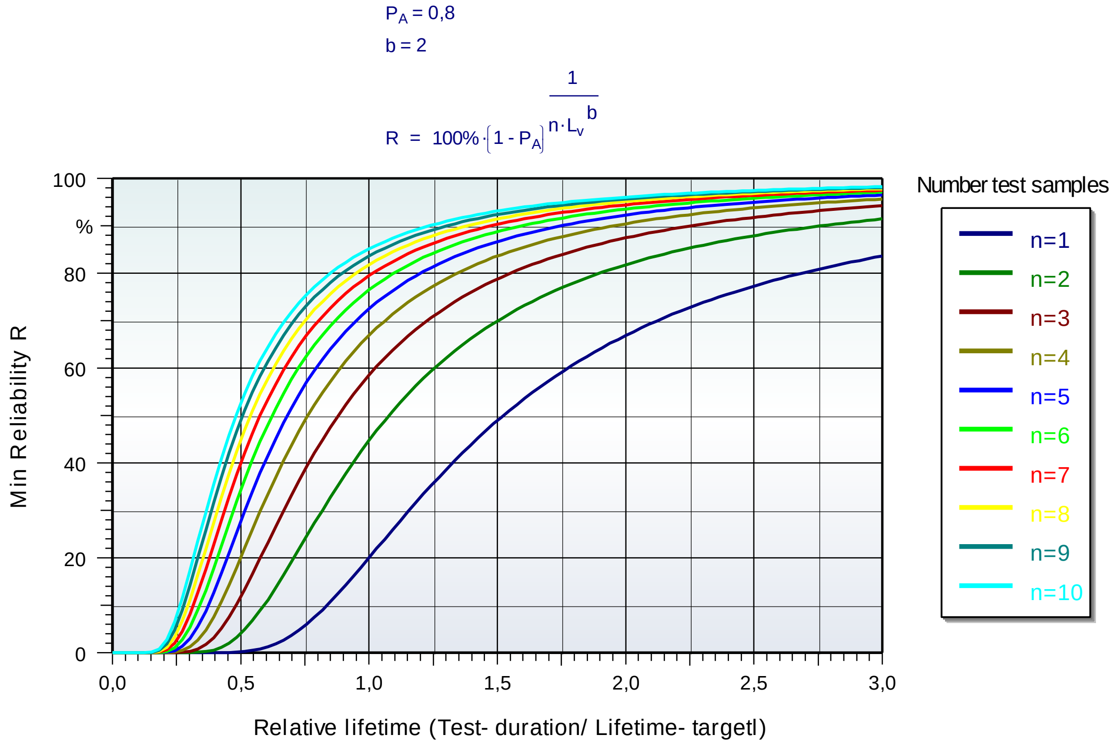
Using Visual-XSel



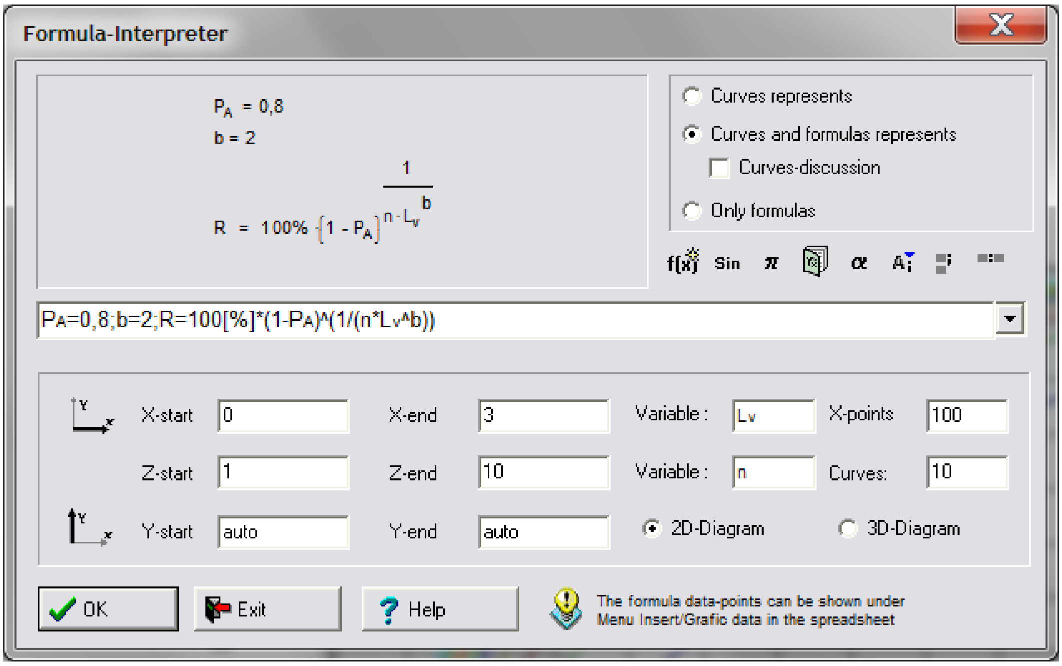
At first time using Weibull a dialog appears for the most important methods:



In some templates no data is needed. For example in the template *LvRb20.vxg* only a formula is represented (see ..\Templates\04_Test_Planning). This is about the determination of a minimum guaranteed reliability in testing with no failures (success run).



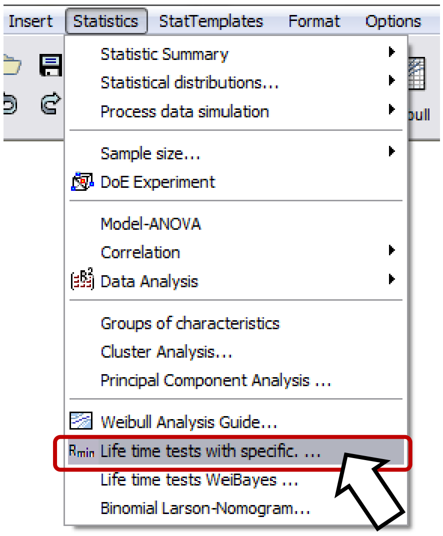
The parameters of the formula can be changed by a double click to the formula in the top of the diagram.
Alternatively the formula can be opened through the menu point *Tools/Formula*

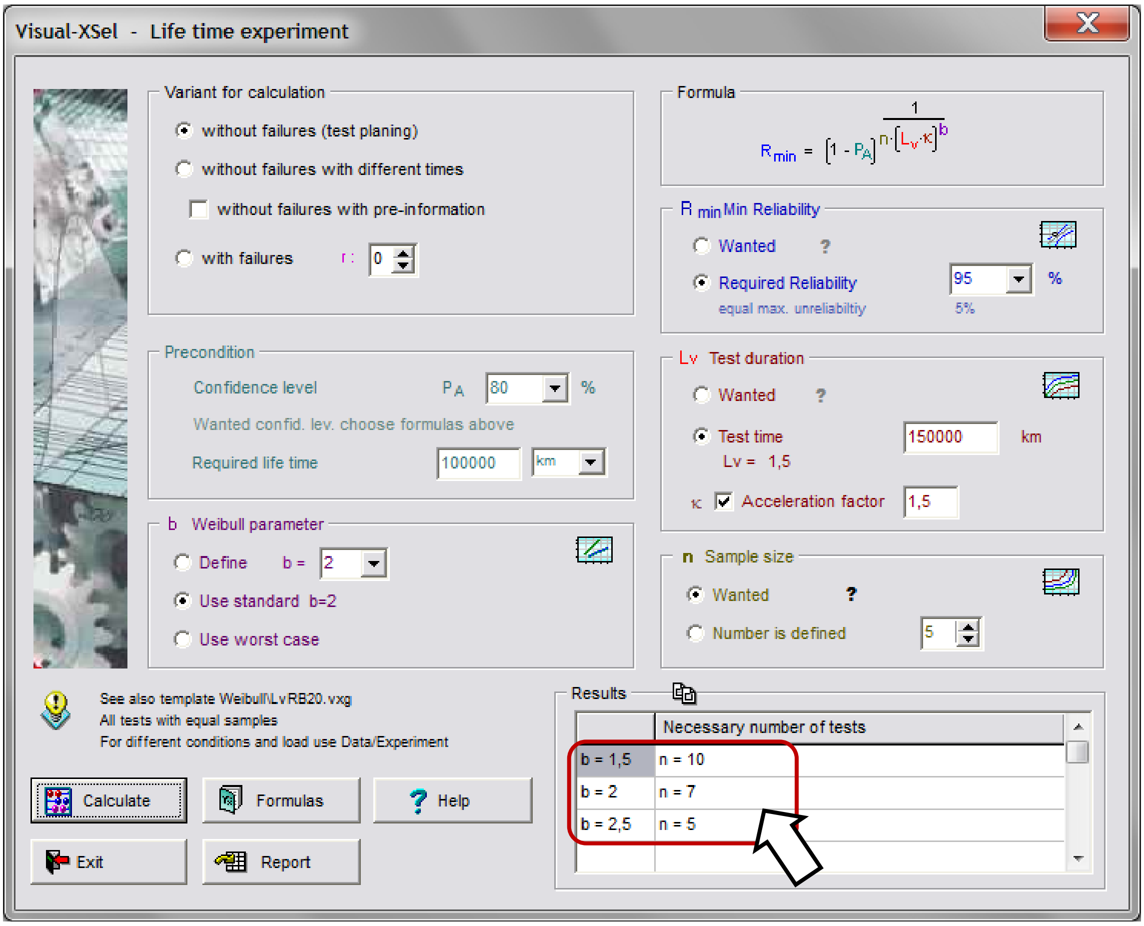


The templates *Weibull_Density_Function.vxg* or *Arrhenius_Model.vxg* are also simple formula charts.

Life time tests – required sample size

Via the main guide or the menu point Statistics in the main window, the required reliability, the necessary test duration or the sampling size can be calculated. For this a default information is the confidence level.
This method is also well known as „Success Run“. Here, however, it is also possible to make calculations with unexpected failures. In the dialog it is recommended to go step by step from the top left to the right bottom.
In the dialog below there is shown an example of a needed sampling size for a required reliability of 95%. The Weibull-parameter b was estimated by b=2. It is shown what was needed if instead b=1.5 or b=2.5 would have been.





This method assumes that a required life time is given. If there is no reference, then the method WeiBayes is an alternative.