Requirement and related topics

The basics of statistics and especially statistical distributions are advantageous for these descriptions. Further topics are:

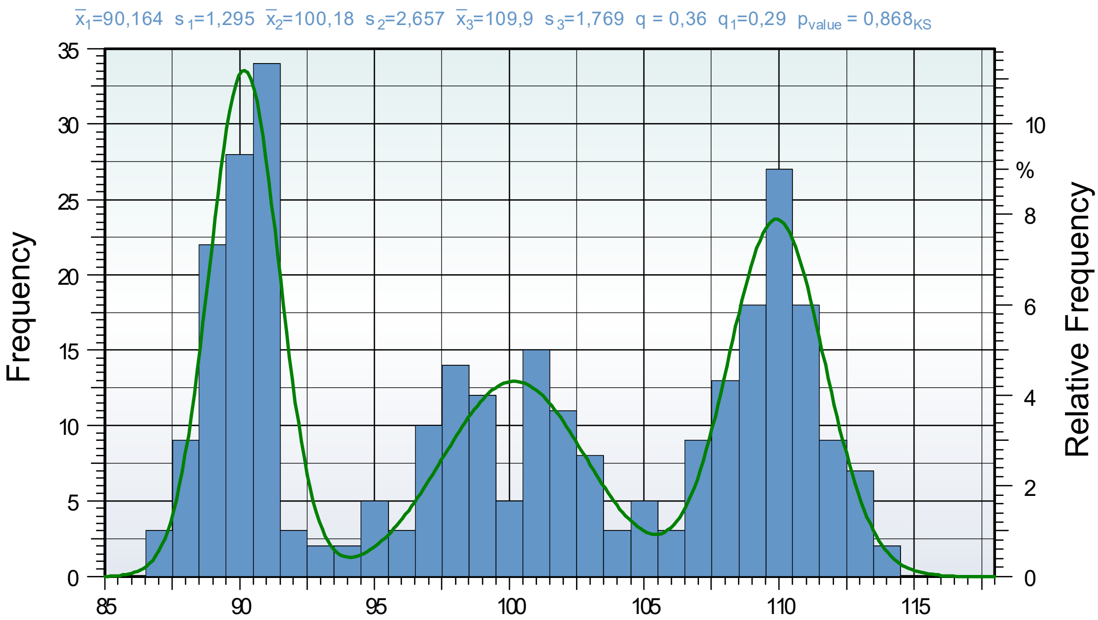
www.weibull.de/COM/Statistics.pdf

www.Weibull.de/COM/Weibull_Analysis.pdf

www.Weibull.de/COM/Process_Capability_Studies.pdf

Introduction

If different distributions overlap, one distribution is no longer sufficient to describe the data situation, one then speaks of a mixed distribution. In the case of a 2-way mixed distribution, this is also called bimodal. Frequent causes are, for example, production batches with different mean values that are mixed up. Ultimately, any number of mixings are possible. The following illustration shows a mixed distribution with at least 3 production batches..



When it comes to service life studies, it is possible that there are different reasons for damage (competing or alternative failure mechanisms).

Purpose and usefulness

Mixed distributions are particularly found when determining a process capability. The use of mixed distributions usually results in better process capability, even if one no longer speaks of a stable process. Overall, the determination of probabilities with the mixed distribution is more precise, because there are always more or less “irregularities”. However, the mixed distribution should only be used if the test for the expected distribution is rejected.

Mixed distribution

Basics

Multi-parameter normal distribution

4-parametric normal distribution

The 4-parametric normal distribution is e.g. used for competing causes of breakage. For example, one cause of breakage is a blowhole and the other a notch effect on the surface. The tests are usually static, e.g. Bending or tensile tests with x = force. Here, the cumulative distribution functions of two normal distributions with their respective mean and standard deviation are combined::

$$H_{(t)} = \Phi_{x;\,\bar{x}1;\,s1} + \Phi_{x;\,\bar{x}2;\,s2} - \Phi_{x;\,\bar{x}1;\,s1} \cdot \Phi_{x;\,x2;\,s2}$$

with $\Phi_{x;\,\bar{x};\,s}$ function for the cumulative normal distribution

The product of both distributions is subtracted again, since it is assumed here that both causes of breakage do not occur at the same time.

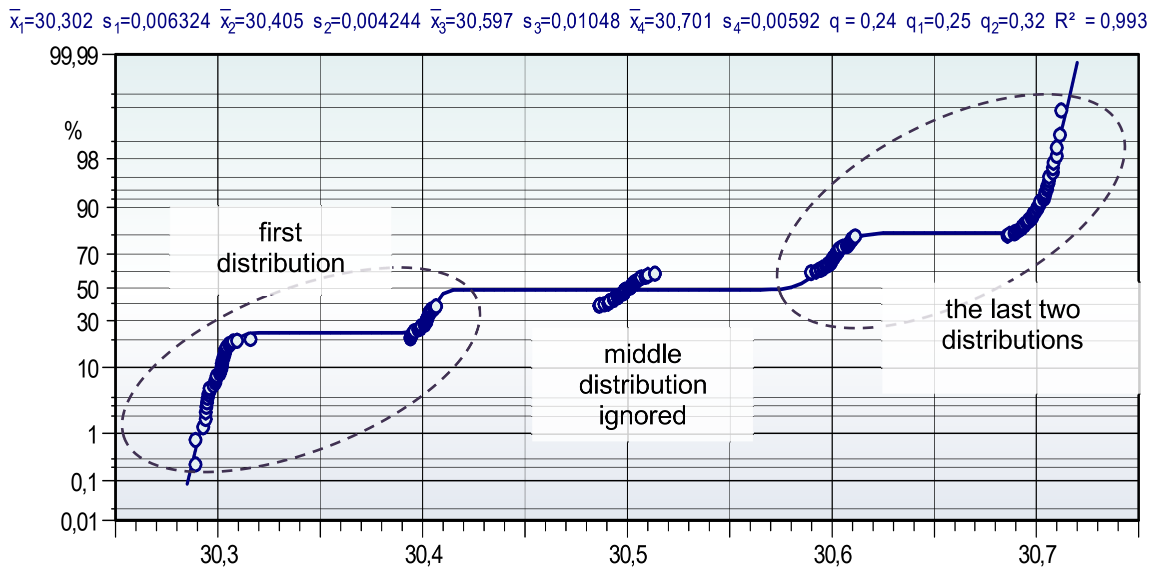
5-parametric normal distribution

A bimodal distribution with alternative causes of failure exists if e.g. two batches with different failure mechanisms are mixed. Under load, each unit fails according to its own mechanism. The proportion of batches 1 and 2 is defined using q and 1-q, respectively:

$$H_{(t)} = q\,\Phi_{x;\,\bar{x}1;\,s1} + (1-q)\,\Phi_{x;\,\bar{x}2;\,s2}$$

As with the 4-parametric normal distribution, the tests are usually static, e.g.
Bending or tensile tests with x = force.

The extension to a 3-times mixed distribution results in an 8-parametric normal distribution etc. Since the calculation of the parameters has to be done iteratively, the calculation of a 5-times mixed distribution at the latest becomes problematic. However, this can occur in production with many mold cavities or parallel production systems. It is therefore suggested here to consider only the edge areas from more than 4 distributions. This is particularly important for the process capability with regard to the tolerance limits::



Mixed distribution

Multi parametric Weibull distribution

4- and 5-parametric Weibull distribution

There are also mixed distributions in the Weibull distribution. Weibull is typically used for lifetime issues. In contrast to static tests, which are based on the normal distribution, dynamic tests are examined here, e.g. bending cycle tests. The mixed distribution arises here from various causes of failure. In the event of two causes of failure, what is known as bimodal distributions are also used here. One differentiates::

1. Competing failure mechanisms
2. Alternative failure mechanisms

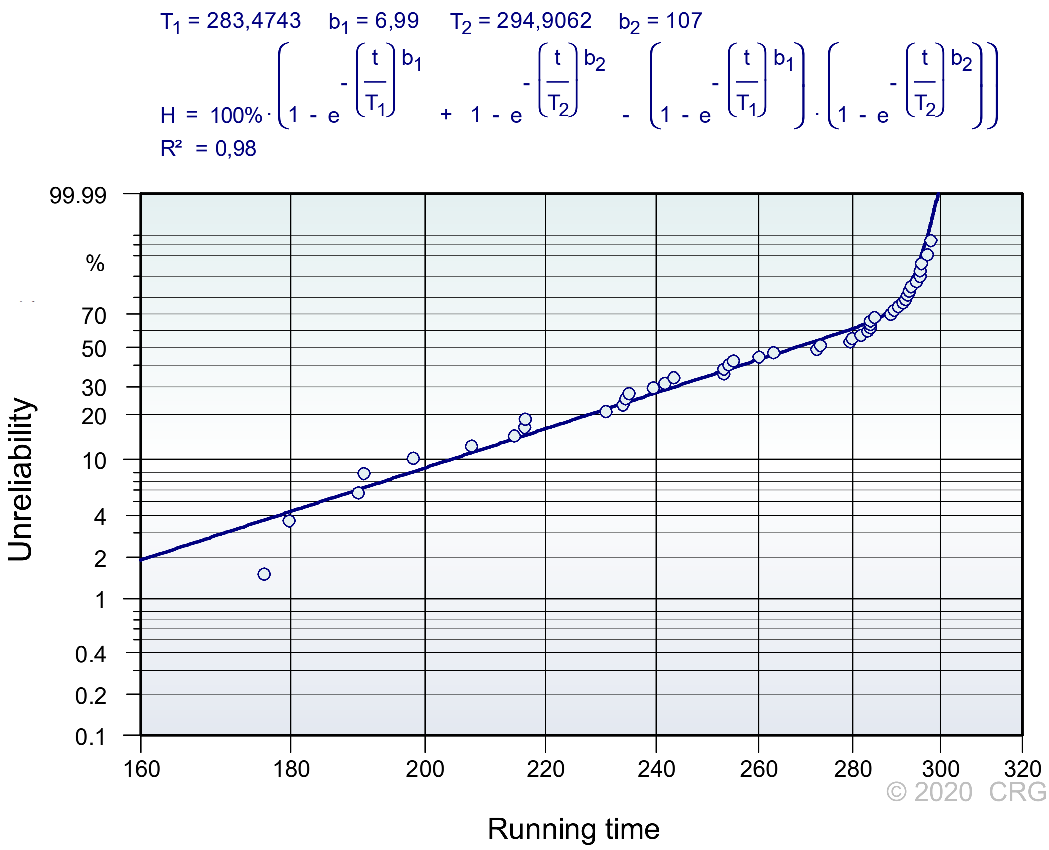
Competing failure mechanisms are assumed if e.g. Different components in one component are defective. The behavior is analogous to the principle of the “weakest link in the chain” with failures at the beginning. The defects of the second component come much later. The first stage is often an early failure, i.e. quality problems caused by production. Competing failure mechanisms are described by the 4-parameter Weibull distribution:

4-parametric Weibull distribution (competing failures)

The corresponding variant in the Weibull distribution is given:

$$\boldsymbol{H_{(t)}} = \left(\mathbf{1} - \boldsymbol{e}^{-\left(\frac{t}{T_1}\right)^{b_1}}\right) + \left(\mathbf{1} - \boldsymbol{e}^{-\left(\frac{t}{T_2}\right)^{b_2}}\right) - \left(\mathbf{1} - \boldsymbol{e}^{-\left(\frac{t}{T_1}\right)^{b_1}}\right) \cdot \left(\mathbf{1} - \boldsymbol{e}^{-\left(\frac{t}{T_2}\right)^{b_2}}\right)$$

For brittle materials (glasses, ceramics, etc.), the Weibull distribution is derived from the theory of minimum extreme value distributions (analogy: weakest link in a chain). Another issue is e.g. the failure of different parts within a system or component.
In the following example there is a clear kink to a steeper slope in the rear area. Due to the initially almost straight initial course, the 3-parametric Weibull distribution cannot adequately cover the points. That speaks for a clear mixed distribution.



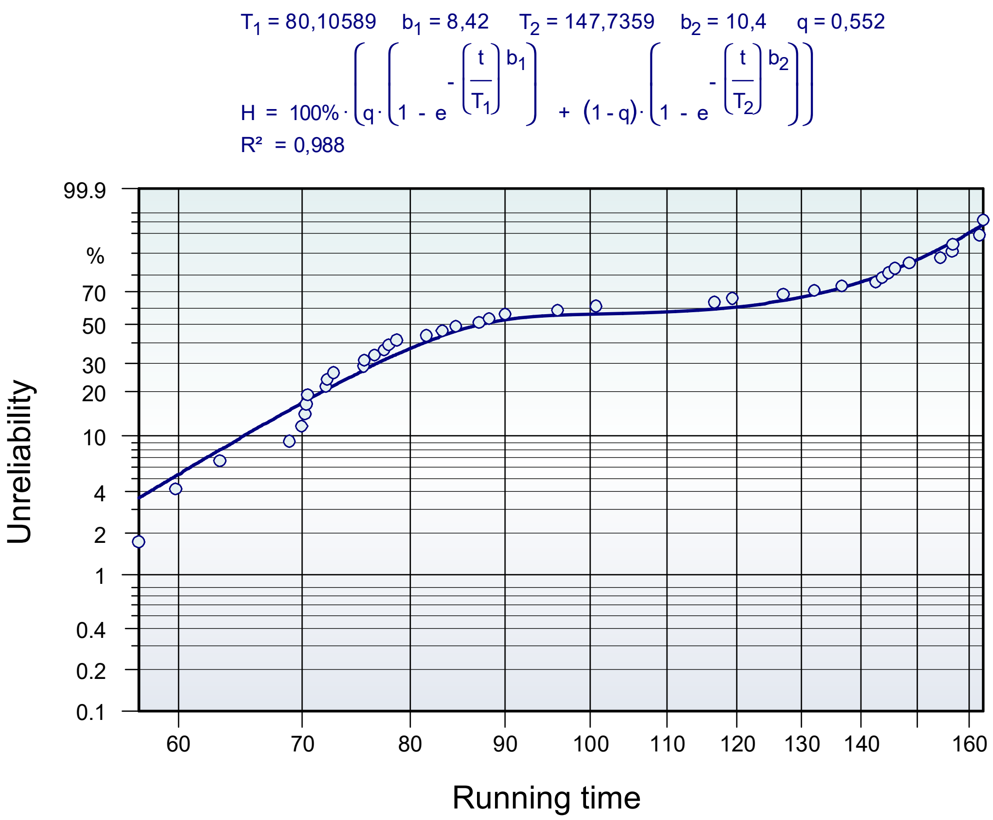
5-parametric Weibull distribution (alternative failures)

Alternative failure mechanisms are used when e.g. one and the same component may even fail with a damage pattern, but the cause for this can be different. Example: A plastic ring in a power transmission breaks because a rubber decoupling is too stiff to dampen the forces or because it is thermally overstressed. It is often not possible to clearly determine whether the component will not be subject to different loads due to different applications (customer behavior).
Alternative failure mechanisms are described by the 5-parametric Weibull distribution, which is the classic form of a mixed distribution::

$$H_{(t)} = q\left(1 - e^{-\left(\frac{t}{T_1}\right)^{b_1}}\right) + (1-q)\left(1 - e^{-\left(\frac{t}{T_2}\right)^{b_2}}\right)$$

q : relative quote of the relevant distribution 0..1

In the following example there is a obvious change from a right to a left curve. This is a distinct indication of a mixed distribution. The first 3 points fall slightly from the front group (3rd cause of failure, e.g. due to previous damage)



The principle of the 5-parameter distribution can also be expanded to 8 parameters and a 3-fold mixed distribution is obtained.

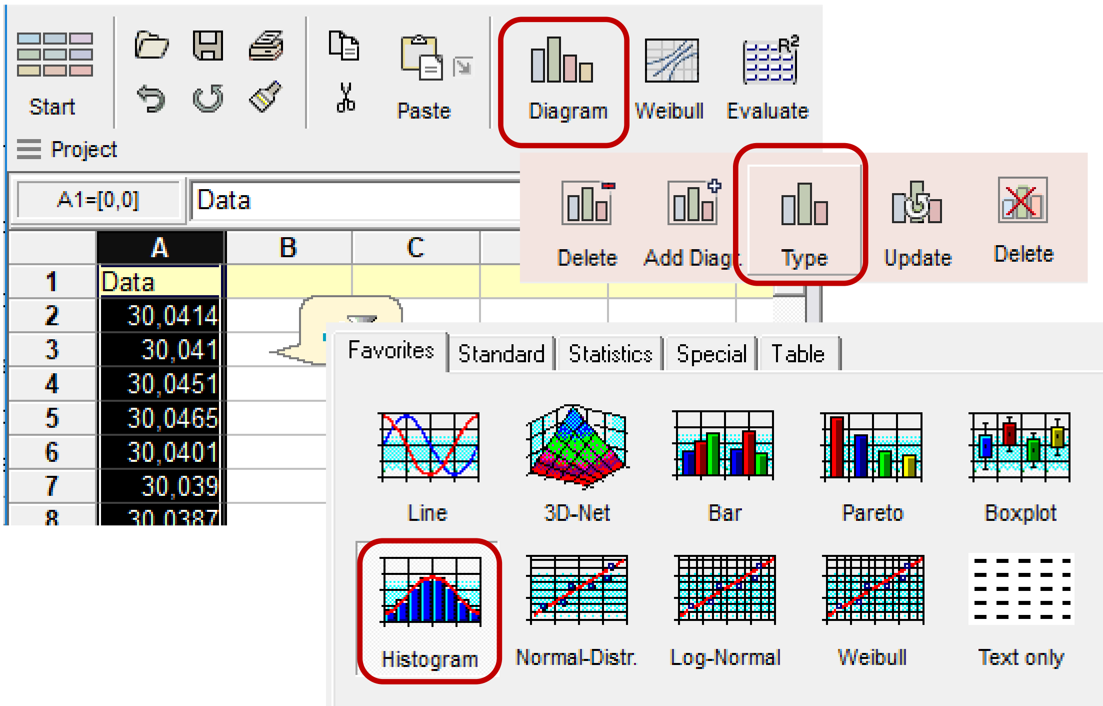
Regardless of the question of competing or alternative causes of failure, if the facts are not known, based on the coefficient of determination, the 4- or 5-parametric form will be chosen. The statistical tests can also provide assistance in choosing one or the other distribution.

The parameters of the Weibull distributions with 4 or more parameters must be solved iteratively. It is possible to optimize using the known method of the smallest deviation squares, or using the max likelihood method. The respective sealing function is partially derived according to the parameters and set to 0. As a starting condition for the iteration, a segment-wise division into two straight line segments with two 2-parametric distributions (each front and back half) is used.

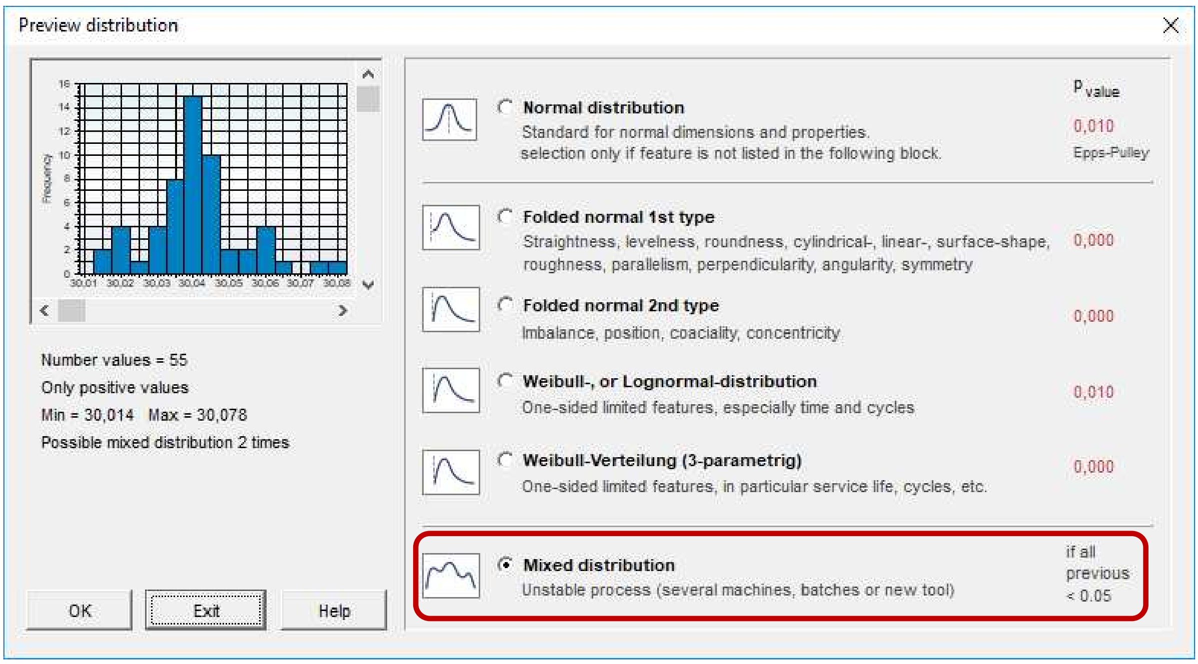
Using Visual-XSel 15.0 / 16.0

www.crgraph.com

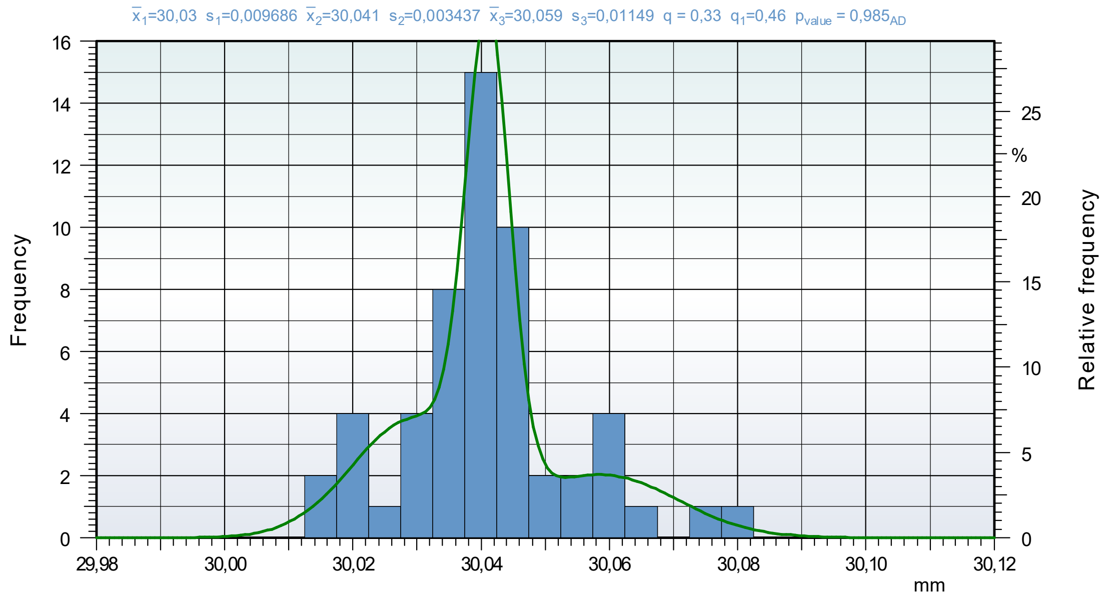
A histogram is shown here as an example of the representation of a mixed distribution. Alternatively, the probability chart (normal distribution) or the Weibull distribution can be selected. Weibull has to be used for lifetime studies.



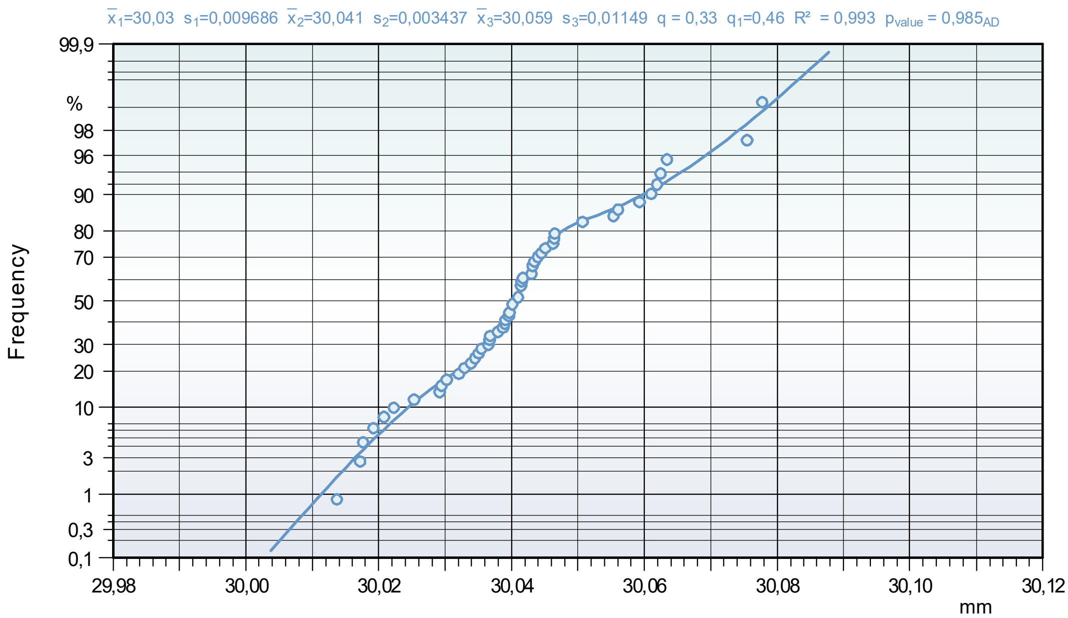
In Version 16 a test distribution test is including. All standard distributions must be rejected because p_{value} < 0.05.



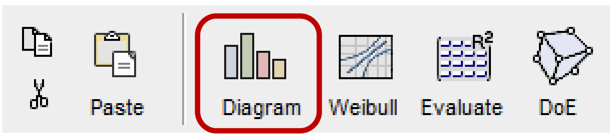
Mixed distribution



As a probability plot with an 8-parametric normal distribution, the same example looks like this:

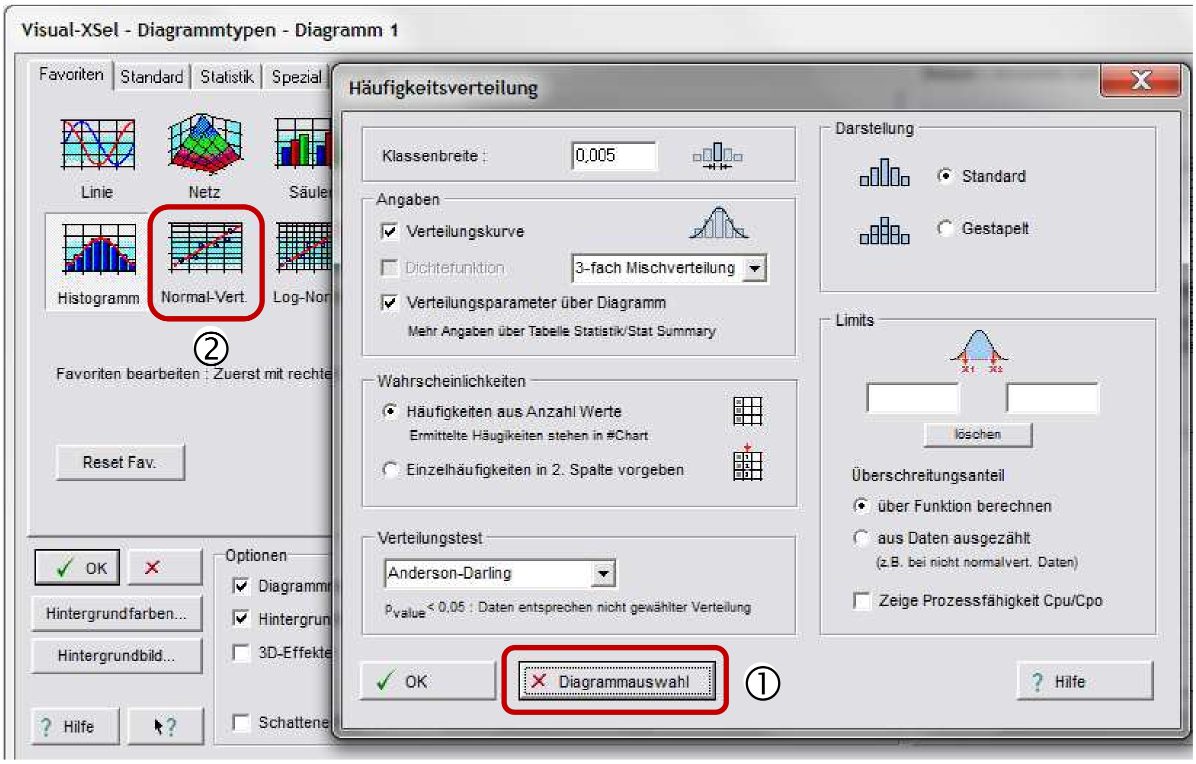


To do this, click the diagram icon again



First, the dialog box for the histogram appears again automatically, which can be closed by pressing the Diagram selection button.

Mixed distribution



Danach wechseln Sie in die Normalverteilung und wählen die entsprechenden Optionen.